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Construction of Knowledge Spaces for Problem Solving in Chess — Two Experimental Investigations

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Abstract

Two experimental investigations of elementary chess knowledge are reported. The investigations are based on an extension of the theory of knowledge structures introduced by Doignon and Falmagne (1985). This extension allows the construction of surmise relations with the help of the formal principles “set inclusion” and “sequence inclusion”. In the case of chess the basic units for problem construction are the tactical elements of the game — the “motives”. In terms of problem solving, these motives can also be seen as subgoals for the problems’ solutions. The results of the two experiments show the importance of the principle of “sequence inclusion” which states that the surmise relation depends on the sequence of motives within a problem. The investigations also demonstrate the suitability of the theory of knowledge structures for testing psychological theories.
1 Introduction

Chess involves one of the most complex and demanding knowledge domains. Not only the game of chess itself, but also the construction of problems which are adequate for an efficient assessment of knowledge concerning the tactical elements of chess, for example, requires a large amount of knowledge and experience and — as we will show here — some special technical principles.

Both of the reported experiments are based on the theory of "knowledge-spaces" introduced by Doignon and Falmagne (1985). This theoretical approach provides the framework for our considerations concerning the ordering of a set of chess problems and the hypothetical structure of knowledge referring to the tactical elements of chess.

The construction of the problems for Experiment I was based on the technical principle of "set inclusion". The theoretical background for the problem construction for Experiment II is more elaborate and in some respects more "realistic". In Experiment II the sequence of tactical elements within a chess problem is taken into account. Additionally, an "order of difficulty" on the set of tactical elements may be assumed. These concepts will be presented in this paper.

First we will describe some basic elements of the theory of knowledge spaces (Doignon & Falmagne, 1985 and Falmagne, Koppen, Johannesen, Villano, Doignon, 1991). One element of this theory is the "surmise relation". This (reflexive and transitive) relation is defined on a set of problems which are to be solved by a population of subjects. If there is, for example, a problem set Q which consists of problems a, b, c and d, the expression \( a \succeq c \) denotes the following:

If a subject is able to solve problem a, then this subject will also be able to solve problem c.

"\( \succeq \)" is the surmise relation and can be depicted as a Hasse diagram (see Figure 1). The set of problems which can be answered correctly by a subject, is called the knowledge state of that subject. From a surmise relation all possible knowledge states can be inferred. The set of all possible states — with respect to a surmise relation — is called a (quasi-ordinal) knowledge space.

A crucial problem of the theory of knowledge spaces is the establishment of a surmise relation. This may be done in several ways. First, there is the possibility of querying experts. Questioning procedures have been developed by Müller (1988), Dowling (1991a, submitted) and Koppen and

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1 This principle is also reported in Albert and Held (submitted).
2 Note: These procedures are not restricted to the establishment of only quasi-ordinal knowledge spaces.

A third approach — which was used in our investigations — is the inference of the surmise relation from the systematically constructed problems themselves, whereby the problems' components are the constituent factors of the surmise relation. This ordering method will be described below.

Additionally, some remarks concerning chess problems, i.e. the tactical elements which are fundamental to our problem sets, will be made.

What types of problems are suitable for an assessment of chess knowledge? For which topics within the large domain of chess playing is experimental exploration feasible? Because of the extreme complexity of chess we must restrict ourselves to problems for which a unique set of possible solutions can be found. In addition we must be able to describe the knowledge necessary for solving the problems.

Problems which are likely to fulfill these requirements are tactical chess problems such as those found in newspapers. Two typical problems of this kind are shown in Figure 2. The solution for example problem (a) is: 1. Ne5+ draw. The solution for problem (b) is: 1. Qe7+ Qe7; 2. Bd6 Qd6; 3. Re8 mate. The solving of such problems requires the knowledge of tactical elements which can be used for a classification of the problems' difficulty with respect to the other elements of the problem set.

These tactical elements are commonly called "motives". The motives considered in our investigations are "pin", "guidance", "deflection", "fork", "stalemate", "elimination", "clearing" and "promotion".

Problem (a) of Figure 2 contains the motive "fork", problem (b) contains "deflection", "guidance" and "pin". Definitions and examples for all motives are given in Appendix A.

As shown in Figure 2, a problem may contain more than one motive — a large variety of combinations is possible. One motive can appear more than once within a problem.

Thus chess problems can be viewed as being combinations of motives.
Figure 2: Two examples for tactical chess problems.

There are probably other factors involved in a chess problem which also have an influence on the solution process\(^3\). Motives surely are elements of that knowledge a person must possess if he or she is to be capable of solving the type of problems used in our investigations.

Next we will introduce the theoretical considerations underlying our investigations. We will show in which ways surmise relations may be established on sets of component based chess problems. These surmise relations are inferred from several psychological assumptions which will also be outlined below. The relations, i.e. the sets of possible knowledge states constitute the hypotheses of our investigations.

The empirical test of the hypotheses will therefore mainly consist of a comparison between the sets of theoretically inferred possible knowledge states and the sets of states which occur as a result of the experiments.

\(^3\)These factors are not the subject of our investigation; considerations regarding these factors can be found in the general discussion.
2 Theory

In this section we will describe two principles for the establishment of a surmise relation on sets of chess problems. Both principles are based on the concept that the difficulty of a chess problem depends mainly on the motives which a subject must know in order to find the correct solution of the problem.

We assume a finite set $M$ of motives. Let the problem space $P(M)$ be the set of all chess problems which could, in principle, be solved with only the knowledge of the motives out of $M$.

**PRINCIPLE 1** (set inclusion)

For a chess problem $p$ let $F(p)$ be the set of the motives which a subject must know in order to find a correct solution for $p$. We assume that the difficulty of a chess problem $p$ depends only on $F(p)$. For the establishment of a surmise relation on $P(M)$ we can therefore identify a problem $p \in P(M)$ with the set $F(p) \subseteq M$ of motives which must be known by a subject in order for him or her to find the solution of $p$.

We define a surmise relation $\preceq$ for all problems $p, q \in P(M)$ through the following condition:

$$p \preceq q :\iff F(p) \subseteq F(q)$$

That means, if a person is able to solve a problem $q$ then he or she is also able to solve all problems which could be solved with the knowledge of a subset of $F(q)$. The surmise relation $\preceq$ on $P(M)$ is therefore defined through a partial order on the power set of $M$.

**Example**: Let $M := \{a, b, c\}$. Figure 3 shows the resulting surmise relation.

**PRINCIPLE 2** (sequence inclusion)

In Principle 1 the order in which the motives occur in the solution of a chess problem does not influence the resulting surmise relation. To ignore this order, however, is problematic for chess problems, because the motives do not occur independently.

In Principle 2 the order in which the motives occur in the solution of a problem plays a central role in the construction of the surmise relation on $P(M)$. The central idea of Principle 2 is that a problem $a$ is more difficult than a problem $b$ if the sequence of motives which must be found by a subject to solve $a$ includes the sequence of motives which must be found to solve $b$. That means we assume the solution process for these types of problems consists of two components. The first component is the
knowledge of motives and the second component is the ability to find the correct sequence of motives which leads to the solution of the problem.

For a chess problem \( p \in P(M) \) let \( G(p) \) be the ordered tuple of motives which occur in the solution of the problem. \( G(p) = (m_1, \ldots, m_k) \) means that the first motive which occurs is \( m_1 \), the second is \( m_2 \) etc. We define:

\[
M_k := \{(m_{i_1}, \ldots, m_{i_k}) \mid m_{i_1}, \ldots, m_{i_k} \in M\}
\]

\[
M_N := \bigcup_{k \in \mathbb{N}} M_k
\]

\( M_k \) is the set of all ordered \( k \)-tuples of motives out of \( M \) and \( M_N \) is the set of all ordered tuples of motives out of \( M \).

It's clear from chess experience that some motives are much easier to detect and process than others. That means that the motives themselves can be ordered with respect to their difficulty. Therefore we can assume a quasi-order \( \sqsubseteq_M \) on \( M \). For motives \( m_i, m_j \in M \) the interpretation of \( m_i \sqsubseteq_M m_j \) is "Every person who knows \( m_i \) also knows \( m_j \)."

We assume that the difficulty of a chess problem \( p \in P(M) \) depends only on \( G(p) \). For the establishment of a surmise relation \( \preceq \) on \( P(M) \) we can therefore identify a problem \( p \) with the ordered tuple \( G(p) \). That means that it is sufficient to define a relation \( \sqsubseteq \) on the set of all ordered tuples of elements out of \( M \). For problems \( p, q \in P(M) \) we then define as in Principle 1:

\[
p \preceq q :\Leftrightarrow G(p) \sqsubseteq G(q)
\]

We will now present two equivalent formalizations of the assumed relation \( \sqsubseteq \) on the set of all ordered tuples of elements out of \( M \). We show both formalizations because each of them provides advantages which could not be united within one single approach. Formalization 1 (problem generation)
consists of a number of weak assumptions which are relatively plausible from the intuitive point of view we described above. It also describes the concept of an inductive definition of the assumed relation. Formalization 2 (problem evaluation) has the advantage of being relatively compact and easy to understand. Further practical advantages of the differentiation between these two approaches can be found in the general discussion. To distinguish between the two formalizations we named the relations which follow from them for the moment as $\sqsubseteq_1$ and $\sqsubseteq_2$.

**Formalization 1 (problem generation):** We establish $\sqsubseteq_1$ in an inductive way. First we make assumptions under which conditions $a \sqsubseteq_1 b$ should hold for a pair $a, b \in M_N$. These conditions are purely syntactical. Then we define the relation $\sqsubseteq_1$, as the set of all pairs $(a, b)$ with $a, b \in M_N$ for which $a \sqsubseteq_1 b$ follows from the following list of assumptions.

1. $\forall m_1, \ldots, m_k \in M \ (m_1, \ldots, m_k) \sqsubseteq_1 (m_1, \ldots, m_k)$
   
   reflexivity

2. $\forall m_1, \ldots, m_k, m_1', \ldots, m_i', m \in M$

   $$(m_1, \ldots, m_k) \sqsubseteq_1 (m_1', \ldots, m_i') \Rightarrow$$

   $$(m_1, \ldots, m_k) \sqsubseteq_1 (m_1', \ldots, m_j', \ldots, m_i', m, m_{j+1}, \ldots, m_i')$$

   insert rule

3. $\forall m_1, \ldots, m_k, m_1', \ldots, m_i, m_i', m'' \in M$

   $$((m_1', \ldots, m_i') \sqsubseteq_1 (m_1'', \ldots, m_k')) \iff$$

   $$((m_1, \ldots, m_j, m_1', \ldots, m_i', m_{j+1}, \ldots, m_k) \sqsubseteq_1$$

   $$\ (m_1, \ldots, m_j, m_1'', \ldots, m_k', m_{j+1}, \ldots, m_k))$$

   sequence insert rule ($\Rightarrow$) and sequence elimination rule ($\Leftarrow$)

4. $\forall m_i, m_j \in M \ (m_i) \sqsubseteq_1 (m_j) \iff m_i \sqsubseteq_M m_j$

   relationship between $\sqsubseteq_1$ and $\sqsubseteq_M$

The relation $\sqsubseteq_1$ is the set of all pairs $(m_i, m_j)$, $m_1, m_j \in M_N$ for which we can prove $m_i \sqsubseteq_1 m_j$ from our assumptions.
Example: Let $M := \{a, b\}$. Figure 4 shows the resulting quasi-order $\subseteq_1$ on $M_1 \cup M_2$. We show as an example how the relation $(a) \subseteq_1 (a, b)$ can

Figure 4: Problem structure based on the assumptions of formalization 1, if we assume $x \subseteq_M y \iff x = y \ (x, y \in \{a, b\})$.

be derived from the assumptions. Because of the reflexivity of $\subseteq_M$ we have $a \subseteq_M a$ and using assumption 4 we can conclude $(a) \subseteq_1 (a)$. Assumption 2 shows that we can insert $b$. So we indeed have $(a) \subseteq_1 (a, b)$.

From assumption 4 it follows that the resulting partial order $\subseteq_1$ depends largely on the relation $\subseteq_M$ on $M$. The following example shows this dependency.

Example: Let $M := \{a, b\}$. Figure 5 shows the resulting quasi-order $\subseteq_1$ if we assume $x \subseteq_M y$ if and only if $(x = y \vee (x = a \land y = b))$ for $x, y \in \{a, b\}$.

Figure 5: Solid lines show the resulting quasi-order $\subseteq_1$, if we assume $x \subseteq_M y \iff x = y \ (x, y \in \{a, b\})$. Dotted lines show the additional pairs, if we assume $x \subseteq_M y \iff x = y \vee (x = a \land y = b)$, whereby $x, y \in \{a, b\}$.
Formalization 2 (problem evaluation): We define a relation $\subseteq_2$ on $M_N$.

**Definition:** Let $(m_1, \ldots, m_k), (m'_1, \ldots, m'_l) \in M_N$. Then $(m_1, \ldots, m_k) \subseteq_2 (m'_1, \ldots, m'_l)$ if and only if there exists a function $f : \{1, \ldots, k\} \rightarrow \{1, \ldots, l\}$ which fulfills the following conditions:

1. $\forall i, j \in \{1, \ldots, k\} \quad (i < j \rightarrow f(i) < f(j))$
2. $\forall j \in \{1, \ldots, k\} \quad m_j \sqsubseteq_M m_{f(j)}$

Now we show the equivalence of the relations $\subseteq_1$ and $\subseteq_2$.

**Theorem 1:** For all $(m_1, \ldots, m_k), (m'_1, \ldots, m'_l) \in M_N$ the following statements are equivalent:

1. $(m_1, \ldots, m_k) \subseteq_1 (m'_1, \ldots, m'_l)$
2. $(m_1, \ldots, m_k) \subseteq_2 (m'_1, \ldots, m'_l)$

A proof of this theorem can be found in appendix B. Because of the equivalence of $\subseteq_1$ and $\subseteq_2$ we set $\subseteq := \subseteq_1 = \subseteq_2$.

**Theorem 2:** $\subseteq$ is a quasi-order on $M_N$.

For a proof of the theorem see also Appendix B.

It must be pointed out that it is impossible to test our assumptions concerning $\subseteq$ directly. However, we can test the surmise relation $\preceq$ on $P(M)$ which results from these assumptions. This approach is described in Experiment II.

**Experiment I**

Problem construction and problem ordering for this first experiment are based on “Principle 1” outlined above. The order on the problem space $P(M)$ is therefore established solely by means of set inclusion.

The set of motives $M$ contains the four motives $F$, $P$, $G$ and $D$. The complete combination of these motives (the power set of $M$) yields — without the “empty problem” — 15 different motive sets (see Figure 6).

We assume that if a problem $p$, which is constructed from the motives $F(p)$ is solved correctly, then all problems $n$ which are constructed from motive sets $F(n)$, being subsets of $F(p)$, will also be solved.

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4 We are grateful to Mathieu Koppen from the Katholieke Universiteit Nijmegen (The Netherlands) for his invaluable comments on an earlier draft of this paper. These comments showed us that it also would be useful also to introduce Formalization 2.

5 We would like to thank B. Hierholz, who both constructed the problems and conducted the experiment.
The main purpose of this experiment is to test the assumed relation on the problem space by a comparison of the theoretical and the empirical knowledge states. The hypothetical relation on the problem space is depicted in Figure 6 as a Hasse diagram.

Figure 6: Hypothetical problem structure of Experiment I.

METHOD

Problems  The motives used for the problems are fork ($F$), pin ($P$), guidance ($G$) and deflection ($D$). For each of the 15 motive sets one possible realization as problem was chosen. Four of the problems were taken from literature (see Appendix C.1), all other problems were constructed specifically for the investigation. A complete list of the problems (including the solutions) can be found in Appendix C.1.

Generally two different types of solution to the problems are possible: (a) White has to achieve a “mate in three moves” and (b) White has to reach a winning position within three moves.

Subjects  Thirteen male subjects, ranging in age from 19 – 54 years took part in the investigation. All subjects were members of chess-clubs and therefore familiar with the rules of the game. Nevertheless their chess playing ability varied widely.

Procedure  The experiment was conducted at the Chess-Club Ladeburg, Fed. Rep. of Germany. It was announced as a club competition and took place on a usual meeting date of the club. The experiment was conducted in one room in a setting familiar to the subjects.

First, the subjects were given written instructions for the experimental procedure. The subjects were asked to solve the problems “as accurately
and quickly as possible.” Additionally, they were instructed on how to write down the solutions (as three consecutive moves).

Each problem is printed on a single card as a diagram (see Figure 2). Information about the type of problem is printed (“mate in three moves” or “winning position in three moves”) below the diagram. The time, needed for the solution is controlled by the subjects themselves with the aid of a chess clock. The subjects were asked to note this time on each problem card.

The problems were presented in the order of their hypothesized difficulty, so the problem with four motives was the first and the problems with one motive were the last to be presented. This sequence of presentation was chosen on the basis of assumptions regarding “warming up effects” and fatigue. The possibility that the subjects “learn” to recognize the motives during the investigation by solving the simpler problems first was to be kept to a minimum. It was also assumed that fatigue would not severely influence the solving of the simple (one motive) problems, which were presented last.

After finishing a problem the subjects returned the diagram to the experimenter; the next problem was then handed to them.

RESULTS

Table 1 shows the results for the 13 subjects, where “+” denotes a correct and “-” an incorrect answer. Subjects with a response pattern which is consistent with our hypothesized knowledge space are marked with an asterisk.

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Table 1: Results Experiment I: correct and incorrect answers

We see that the response patterns of only four subjects (2, 3, 4, 5) agree with our hypothesis. Subjects 7 and 10 each provide inconsistencies for only
one problem. Column “d” shows the symmetric difference\(^6\) of each person's response pattern from the closest hypothetical state. Figure 7 (shaded nodes denote incorrect answers) shows the results for both an inconsistent (top) and a consistent subject (bottom). In Appendix D a summary of

![Diagram](image)

**Figure 7:** Examples for results of Experiment I. Problems which were answered incorrectly are marked by shaded nodes. The structure on the top shows a result which does not coincide with our hypothesis. The lower structure demonstrates a result which is consistent with respect to our hypothesis.

all solution times is provided. We can see that the total time needed by the subjects for solving the problems ranges from 38.5 minutes to 3 hours. A large variation within the times that were needed for solving the single problems can also be observed.

\(^6\)The symmetric difference \(d\) between two sets \(A\) and \(B\) is defined as follows:
\[d(A, B) = |A \Delta B|,\] where \(A \Delta B = (A \setminus B) \cup (B \setminus A)\).
DISCUSSION

For summary the results of Experiment I are not satisfactory, since only four of the observered response patterns are elements of the predicted knowledge space. The reasons for this result may both be found in the experimental design and the theoretical approach of "Principle 1".

First we will discuss the problems concerning the experimental design. The main problems here may be the experimental setting as a group experiment and the missing limit on solution times. As a consequence, some subjects had already completed all of the problems, while others were still working on them. This may have caused motivational difficulties. As we can see from the table in Appendix D, subject 10 finished the investigation about two and a half hours later than subject 2.

The order of problem presentation (in order of decreasing difficulty), obviously did not have the intended effects because especially for the subjects with very long solving times, motivational decrease and fatigue may have prevented the correct solution of the simple problems.

Another critical item is the inhomogeneity of the problem set. Some of the problems are typical representatives of "problem chess", some of them are "endgame studies" and others are similar to the positions of practical chess games. Because average chess players are usually not as familiar with "problem chess" and "endgame studies", this inhomogeneity may have influenced solving behavior.

As we have already mentioned (see "Principle 2" above), ignoring the sequence of the motives within the problems seems to be problematic for chess problems.

Experiment II

The second experiment makes use of the extended ordering method of "Principle 2". Therefore, not only the existence of the motives, but also their sequence within the problem is considered. Six different motives were used for problem construction. They are designated as $F$, $S$, $E$, $C$, $G$ and $T$, so the set of motives is $M = \{F, S, E, C, G, T\}$. Each problem contains four motives at the most.

We distinguish between two different types of hypothetical problem structures. If we assume that none of the motives can be detected or be processed more easily than other motives (i.e. $m_i \sqsubseteq_M m_j \iff i = j$), the resulting structure consists of two non-connected structures as shown in Figure 8. Both the entire structure and the single sub-structures may be tested empirically, because they are both compatible with our theoretical assumptions of "Principle 2".
If we assume that Motive $F$ can be detected and dealt with more easily than motives $S, E, C, G$ and $T$ (i.e. $m_i \subseteq m_j \Leftrightarrow i = j \lor (m_i = F \land m_j \in \{S, E, C, G, T\})$, a hypothetical order as shown in Figure 9 results. This further assumption adds some additional pairs of problems to the surmise relation (e.g. $(G, S) \succeq (G, F)$).

Figure 8: Hypothetical problem structure of Experiment II, based on the assumption that none of the motives is more difficult to detect or process than any other motive out of our motive set.

**METHOD**

**Problems** The problem set consists of 17 problems\(^7\) constructed from the motives fork ($F$), stalemate ($S$), elimination ($E$), clearing ($C$), guidance ($G$) and promotion ($T$). The number of moves necessary for the solution of the problems ranges from one to four, whereby one move does not necessarily represent one motive.

In all problems, White moves first, whereby the optimal moves do not necessarily lead to a mate. Forcing a stalemate or reaching a winning position can also be optimal solutions. A complete list of the problems is provided in Appendix C.2.

**Subjects** The experiment was conducted with 37 male and 9 female subjects recruited through an announcement in the local newspaper. Their ages ranged from 14 to 71 years. All of them were familiar with the basic rules of chess. For taking part in this investigation each subject was paid DM 12.-

\(^7\)Due to an error in the presentation of the problem containing motives $E$ and $S$, we cannot use problem $(E, S)$ for further analysis. So only 16 problems remain — as shown in Figures 8 and 9 for the results.
Figure 9: Hypothetical problem structure of Experiment II, based on the assumption that motive F is to detect and process easier than all other motives out of our motive set.

**Apparatus** The experiment was run on SUN-3 Workstations with monochrome 19" monitors. The problems were presented on the screen and the subjects are required to make their moves using an optical mouse. The graphical chess surface is provided by the public-domain program "nchess". The entire experimental procedure is running under control of the hypermedia-system "KMS" (see Scribe Systems, 1988 and Aksycyn, McCracken & Yoder, 1988). A KMS Action-Language program invokes nchess, presents the problems in a randomized order and produces protocols of the subjects’ answers and the times, needed for solving the problems. Also all instruction texts are presented in KMS.

For “playing against the subjects”, a small chess program has been written, which is able to communicate with the nchess surface.

**Procedure** The subject is asked to sit down at the computer screen, then she or he is told that the mouse is the only device, which he or she is allowed

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8"nchess" was written by T. Anderson in 1988. It is available via ftp at "comp.sources.games".

9The software was written by J. Unnewehr, M. Kadijk, S. Fries and R. Stökl at the University of Heidelberg in 1990.
to handle during the experiment. The subject is also asked to use only the left mouse button.

A run of the experiment consists of two main phases: a training phase and an experimental phase. The training phase allows the subjects to learn and practise moving the pieces on the computer’s chess board. During this procedure, the subject is required — while playing the white pieces — to capture a black piece. There are five different training tasks to be solved. If the subject makes only one mistake while solving the training tasks, a new sequence of the same tasks is started. The training phase is terminated as soon as a sequence of all five problems has been solved correctly. If the subject fails three runs of the training phase, the whole experimental run is terminated.

Upon completion of the training phase, the instructions for the experimental phase are presented on the screen. In the following we will provide the most important parts of the instructions\textsuperscript{10}. The complete text can be found in Appendix E.

"[...] For every problem a chess board will be displayed on the right side of the screen. You will always play the white pieces, which move from ‘the bottom to the top of the screen’. White moves first.

Initially the position will be presented for 90 seconds without providing the opportunity to move. During this time, please think about the move or the combination of moves which lead — with as few moves as possible — to the best result for White.

After these 90 seconds [...] you can make your move. Please move within 30 seconds. If more than one move is necessary, the computer will immediately answer with Black’s move. [...]"

As soon as the problem is finished, the chess program will be terminated and a new problem will be started. Altogether there are 20 problems.

Illegal moves will not be accepted by the chess program. In case of a pawn promotion, you will be asked to which figure the pawn is to be promoted. [...] Please select the appropriate piece with the mouse. [...]"

After the subject has read the instructions, the experimentator starts the run. The first problem is displayed. Every experimental run starts with three very simple "dummy problems" (see Appendix C.3), which are not

\textsuperscript{10}The original instructions are written in German. The translation is as close as possible to the original text.
relevant for further analysis. After these initial problems the 17 relevant problems are presented in random order.

The subject now has 90 seconds to work out a solution to the problem. The remaining time is indicated in the upper left-hand corner of the screen. As soon as the 90 seconds are over, a short "beep" is sounded and the chess surface is activated. The subject is now required to make the moves.

As the subjects were told in the instructions, each move must be made within 30 seconds. If this time is exceeded, a message is displayed requesting the subject to draw faster. The time limitation (90 seconds for thinking and 30 seconds to make a move) was chosen because of two possible subjects responses: (1) we wanted to avoid that the subjects start moving too early — without having sufficiently thought about the problem's solution and (2) the subjects should not be given the opportunity to solve the problems while the moving phase is running, because strong variations in thinking times should be avoided.

When a pawn reaches the eighth row, the nchess program automatically displays a Queen on the chess-board. Since this cannot be avoided the program is terminated immediately and a choice menu appears offering the four possible pieces (Queen, Rook, Bishop and Knight); the subject has to select one of them with a mouse-click. If the choice was correct, the game is continued with the selected piece, otherwise it is terminated.

After each of the subjects' moves, the computer will either make a suitable answer move (if the subject's move was correct), or the game will be ended (if the problem is finished or the subject's move was wrong). At no time do subjects receive feedback concerning the correctness of their answers.

**RESULTS**

Figures 10 and 11 show examples for individual results marked within the hypothetical problem structures, whereby shaded nodes denote incorrect answers. In each figure we show a response pattern which matches our hypothesis and a response pattern which does not match. First we determine how well the data fit our hypothetical knowledge spaces. The knowledge spaces, which are derived from the hypothetical surmise relations, are labeled $\mathcal{K}_1$, $\mathcal{K}_2$, $\mathcal{K}_{12}$ and $\mathcal{K}_3$. Spaces $\mathcal{K}_1$ and $\mathcal{K}_2$ correspond to the single sub-structures shown in Figure 8, $\mathcal{K}_3$ is the space for the problem structure, which is based on the assumed order on the set of motives (Figures 9 and 11). $\mathcal{K}_{12}$ is derived from the surmise relation of Figure 8 as a whole. This structure is especially important for a comparison with $\mathcal{K}_3$, which may provide information regarding the effects of the assumption, that motive F is to detect and process easier than all other motives.

Table 2 provides an overview of the numbers of possible states (car-
Figure 10: Examples for results of Experiment II. Problems which have been answered incorrectly, are marked by shaded nodes. The upper diagram shows a result, which doesn’t agree with our hypothesis; the lower diagram shows a result, which does agree.

... and the number of matching and non-matching states within the data; “negative states” are subsets of the problem set which do not belong to a hypothetical knowledge space. In this table, we show the number of subjects with a positive response pattern, a negative response pattern as well as the number of different positive and negative response patterns within the data.

The table in Appendix F gives a complete overview of the subjects’ individual results, including the symmetric differences between observed and hypothetical states. Table 3 shows the total solution frequencies for each problem. Problem \((E, S)\) is in brackets, because it was not used in the analysis.
Figure 11: Examples for results of Experiment II. Problems which have been answered incorrectly, are marked by shaded nodes. The upper diagram shows a result, which doesn’t agree with our hypothesis; the lower diagram shows a result, which does agree.

DISCUSSION

As a whole this experiment can be termed a success. There are, however, a number of violations of our deterministic hypothetical assumptions. In addition the influences of “lucky guesses” and “careless errors” (see Falmagne & Doignon, 1988), which may occur during every diagnostic procedure, must be taken into account. Because of this fact, the assumption that every subject’s response pattern will be an element of the hypothetical knowledge space is unrealistic\(^\text{11}\)

Furthermore, we can see from the table in Appendix F that the symmet-

\(^{11}\)Example: if we assume that the probability of both careless errors and lucky guesses is 0.05, then the probability that the response pattern of the subject is equal to the subject’s knowledge state is $0.95^{10} \approx 0.44$.\]
Table 2: Possible states and matching response patterns for Exp. II

<table>
<thead>
<tr>
<th>Knowl.-Space</th>
<th>Number of pos. states</th>
<th>Number of neg. states</th>
<th>Obs. number of subjects with pos. patterns</th>
<th>Obs. number of subjects with neg. patterns</th>
<th>Obs. number of different pos. patterns</th>
<th>Obs. number of different neg. patterns</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mathcal{K}_1$</td>
<td>41</td>
<td>215</td>
<td>33</td>
<td>11</td>
<td>14</td>
<td>4</td>
</tr>
<tr>
<td>$\mathcal{K}_2$</td>
<td>25</td>
<td>231</td>
<td>37</td>
<td>9</td>
<td>13</td>
<td>7</td>
</tr>
<tr>
<td>$\mathcal{K}_{(12)}$</td>
<td>1025</td>
<td>64511</td>
<td>28</td>
<td>18</td>
<td>20</td>
<td>16</td>
</tr>
<tr>
<td>$\mathcal{K}_3$</td>
<td>368</td>
<td>65168</td>
<td>23</td>
<td>23</td>
<td>15</td>
<td>21</td>
</tr>
</tbody>
</table>

Table 3: Solution frequencies for Exp. II

<table>
<thead>
<tr>
<th>Problem</th>
<th>Solution frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S$</td>
<td>31</td>
</tr>
<tr>
<td>GS</td>
<td>15</td>
</tr>
<tr>
<td>EGS</td>
<td>13</td>
</tr>
<tr>
<td>EEGS</td>
<td>13</td>
</tr>
<tr>
<td>CS</td>
<td>21</td>
</tr>
<tr>
<td>GCS</td>
<td>9</td>
</tr>
<tr>
<td>TS</td>
<td>21</td>
</tr>
<tr>
<td>GES</td>
<td>11</td>
</tr>
<tr>
<td>(ES)</td>
<td>42</td>
</tr>
</tbody>
</table>

Table 3: Solution frequencies for Exp. II

......

ric differences between the response patterns which do not fit our hypothesis and the closest theoretical states is in almost all cases rather small ($\leq 2$).

While analyzing the results, we should pay special attention to the proportions between the hypothetical number of positive states and the hypothetical number of negative states on the one side and the proportions between the observed number of positive response patterns and the observed number of negative response patterns on the other (see Appendix F). We see that our assumptions seem to be adequate for chess problems. The additional assumption that the motive "fork" is to detect and process easier than all other motives makes the problem structure stricter. It therefore comes as no surprise that this assumption leads to a larger number of response patterns which are not consistent with the hypothetical knowledge structure.

In general, we can state that the proposed surmise relations which follow from the assumptions of "Principle 2" have proven suitable for our set of chess problems.

**GENERAL DISCUSSION**

The two experiments we presented — though they refer to the same knowledge domain — differ in the underlying theory, the experimental setting and the "quality" of the results.

The results of Experiment I lead to the assumption that pure set inclusion as introduced in "Principle 1", seems to be — at least with respect
to our experimental conditions — inappropriate for the ordering of component based chess problems. To confirm this assumption, we established an order on the problems of Experiment II by means of “Principle 1”. The resulting order is shown in Figure 12 as a Hasse diagram. The corresponding knowledge space consists of 85 states, the number of negative states is $65452^{12}$. The response patterns of only seven subjects (out of 46) are elements of this knowledge space. Four different positive response patterns and 32 different negative response patterns are observed. It was also shown that for the problems of Experiment II an ordering based on Principle 1 did lead to particularly negative results. Nevertheless, the improved experimental setting and the greater homogeneity of the problem set may also be responsible for the positive results of Experiment II.

Another item which should be discussed, is the suitability of motives for the characterization of chess problems. First, motives may only be adequate as “exclusive” components of simple problems as used in our investigations. However we are aware of the fact that other elements of the problems may also influence their difficulty. In Experiment II we endeavored to minimize

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$^{12}$For a comparison of the respective number of positive and negative states to the other hypothetical knowledge spaces, see Table 2
these influences striving for maximal homogeneity of the problem set. Secondly, for more complex problems, motives will also belong to the set of relevant problem components, but other factors which are indispensable for the knowledge of a good chess player will become more important. More information concerning these factors can be gained from the investigations of de Groot (1965).

Our considerations are based on the theory of Doignon and Falmagne (1985). But the theoretical approach, we introduced here, is also an extension of this theory. We provide methods for the establishment of surmise relations which are primarily dependent on problem components. The construction of problems is an important aspect of our approach. In contrast to Doignon and Falmagne (1985), who do not consider the properties of the problems themselves, we are particularly interested in those elements of problems which may be fundamental to the surmise relation.

It is evident that on the one hand problems may be constructed from a set of components and on the other hand a given set of problems can be ordered according to an analysis of the components which appear in their solution. Formalization 1 can be considered a construction rule, whereas formalization 2 provides a suitable instrument for the ordering of given problems.

In our investigation, we were able to show that the theory of Doignon and Falmagne (1985) can be used for testing psychological theories. Especially in the area of Psychology of Knowledge, many theoretical considerations can be formulated in terms of this theory. The advantage of this approach is that hypotheses can — with the help of the introduced set theoretic concepts — be stated clearly and tested easily.
References


APPENDIX
A Definitions and Examples for the motives used in the investigations

Overview

<table>
<thead>
<tr>
<th>Motive</th>
<th>Name</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>F</td>
<td>Fork</td>
<td>One piece simultaneously attacks two opposing pieces of higher value.</td>
</tr>
<tr>
<td>P</td>
<td>Pin</td>
<td>A piece prevents an opposing piece from moving.</td>
</tr>
<tr>
<td>G</td>
<td>Guidance</td>
<td>An opposing piece is forced to a disadvantageous square.</td>
</tr>
<tr>
<td>D</td>
<td>Deflection</td>
<td>An opposing piece is forced to leave an important line or square.</td>
</tr>
<tr>
<td>E</td>
<td>Elimination</td>
<td>For provoking a stalemate, for example, the elimination of an own piece is forced.</td>
</tr>
<tr>
<td>C</td>
<td>Clearing</td>
<td>For provoking a stalemate, for example, an important line or square is left.</td>
</tr>
<tr>
<td>T</td>
<td>Promotion</td>
<td>Having moved to the 8th line, a pawn has to be promoted to the most suitable piece (Rook, Knight or Bishop, but not Queen)</td>
</tr>
<tr>
<td>S</td>
<td>Stalemate</td>
<td>A possible stalemate has to be detected and has either to be provoked or avoided.</td>
</tr>
</tbody>
</table>

Examples

Deflection:

![Diagram of a chessboard with examples of deflection](image)

Figure 13: Examples for motives: "deflection"
Construction of Knowledge Spaces for Problem Solving in Chess

1. \( Bc8 \)...
   The black Bishop is forced to leave e6 (deflection), because else 2. \( ... \) Be6; 3. c8Q

1. \( ... \) Bd5
2. Bf5: Bb7
3. Be4 White wins, because the black Bishop is forced to leave b7 and after Be4: White wins easily with c8Q.

Pin:

![Chessboard Diagram]

Figure 14: Examples for motives: “pin”

1. Qf8+ Qe8 The black Bishop cannot move away from e7 (pin) because of Bf6+.
2. Rd1+ Rd7
Elimination, Guidance and Stalemate:

![Chessboard diagram]

Figure 15: Examples for motives: "elimination", "guidance" and "stalemate"

1. \( \text{Ktd5+ } \ldots \) Black is forced to eliminate the Knight because else \( \text{Ktf4} \):

1. \( \ldots \) cd5:
2. \( \text{Bg3 } \ldots \) The black Queen is forced to the disadvantageous square \( g3 \) (guidance).
2. \( \ldots \) Qg3:
   stalemate

Promotion and Fork:

![Chessboard diagram]

Figure 16: Examples for motives: "promotion" and "fork"

1. \( \text{f8Kt+} \) White wins, because the black Queen is won (fork and promotion); \( f8Q \) would be an error because White would only achieve a draw.
Clearing and Stalemate:

Figure 17: Examples for motives: "clearing" and "stalemate"

1. b4+ ... The second line is cleared.
2. ... ab:ep/cb:ep If K arbitrary then it's also stalemate.
   stalemate
B Proofs

B.1 Proof of theorem 1:

First we prove $2 \Rightarrow 1$.

Let $(m_1, \ldots, m_k) \subseteq (m'_1, \ldots, m'_i)$. That is equivalent to the existence of a function $f : \{1, \ldots, k\} \rightarrow \{1, \ldots, l\}$ with $m_i \subseteq m_{f(i)}$ and $i < j \rightarrow f(i) < f(j)$ for all $i, j \in \{1, \ldots, k\}$. That implies immediately $k \leq l$.

We prove by induction on $n$ that for each $n \in \{1, \ldots, k\}$ the relation $(m_1, \ldots, m_n) \subseteq (m'_1, \ldots, m'_{f(n)})$ holds.

Let $n = 1$. From the condition $m_1 \subseteq m_{f(1)}$ and our assumption 4 we can conclude $(m_1) \subseteq (m'_{f(1)})$. Using assumption 2 it follows easily that $(m_1) \subseteq (m'_1, \ldots, m'_{f(1)})$.

Let $n = h$ and the proposition be true for all $n < h$. Therefore we have $(m_1, \ldots, m_{h-1}) \subseteq (m'_1, \ldots, m'_{f(h-1)})$. From $m_h \subseteq m_{f(h)}$ we can conclude that $(m_h) \subseteq (m'_{f(h)})$ as above. Using assumption 3 it follows $(m_1, \ldots, m_h) \subseteq (m'_1, \ldots, m'_{f(h-1)}, m'_{f(h)})$. With assumption 2 we can conclude $(m_1, \ldots, m_h) \subseteq (m'_1, \ldots, m'_{f(h)})$. That means that the proposition is true for all $n \in \{1, \ldots, k\}$.

Therefore $(m_1, \ldots, m_k) \subseteq (m'_1, \ldots, m'_{f(k)})$ and again using assumption 2 we can easily conclude that $(m_1, \ldots, m_k) \subseteq (m'_1, \ldots, m'_i)$. This completes the proof of $2 \Rightarrow 1$.

Now we prove $1 \Rightarrow 2$.

We must show that $(m_1, \ldots, m_k) \subseteq (m'_1, \ldots, m'_i)$ implies the existence of a function $f : \{1, \ldots, k\} \rightarrow \{1, \ldots, l\}$ with $m_i \subseteq m_{f(i)}$ and $i < j \rightarrow f(i) < f(j)$ for all $i, j \in \{1, \ldots, k\}$. We prove that by induction on $l$. First its clear from our assumptions that $(m_1, \ldots, m_k) \subseteq (m'_1, \ldots, m'_i)$ implies $k \leq l$.

Let $l = 1$. Then we have $(m_1) \subseteq (m'_1)$ and therefore its clear that $m_1 \subseteq m'_1$. We define $f : \{1\} \rightarrow \{1\}$ by $f(1) := 1$.

Let $l = n$ and the proposition be true for all $h < n$. Because $(m_1, \ldots, m_k) \subseteq (m'_1, \ldots, m'_i)$ follows from our assumptions there are two possibilities.

The first possibility is that there exists $j \in \{1, \ldots, l\}$ with the property that $(m_1, \ldots, m_k) \subseteq (m'_1, \ldots, m'_{j-1}, m'_{j+1}, \ldots, m'_i)$. That means that the relation $(m_1, \ldots, m_k) \subseteq (m'_1, \ldots, m'_i)$ follows by assumption 2 from the relations between shorter tuples. Now the second tuple has length $l - 1 < n$ and we can conclude the existence of a function $f : \{1, \ldots, k\} \rightarrow \{1, \ldots, j - 1, j + 1, \ldots, l\}$ with $m_i \subseteq m_{f(i)}$ and $i < j \rightarrow f(i) < f(j)$ for all $i, j \in \{1, \ldots, k\}$. But then $f$ as a function from $\{1, \ldots k\}$ to $\{1, \ldots l\}$ fulfills also the required properties.
The second possibility is that there exists \((l_p, \ldots, l_q), (l'_p, \ldots, l'_r)\) with the following properties:

1. \(1 \leq p < q \leq k\)
2. \(1 \leq p < r \leq l\)
3. \(m_p = l_p, \ldots, m_q = l_q\)
4. \(m'_p = l'_p, \ldots, m'_r = l'_r\)
5. \(m_1 = m'_1, \ldots, m_{p-1} = m'_{p-1}, m_{q+1} = m'_{q+1}, \ldots, m_k = m'_i\)

That means that the relation \((m_1, \ldots, m_k) \sqsubseteq (m'_1, \ldots, m'_i)\) follows by assumption 3 from the relations between shorter tuples.

As above we can conclude the existence of \(f : \{p, \ldots, q\} \rightarrow \{p, \ldots, r\}\) with \(m_i \sqsubseteq m'_{f(i)}\) and \(i < j \rightarrow f(i) < f(j)\). Now we define a function \(g : \{1, \ldots, k\} \rightarrow \{1, \ldots, l\}\) through

\[
g(i) := \begin{cases} 
  i & \text{for } i < p \\
  f(i) & \text{for } p \leq i \leq q \\
  r + i & \text{for } q < i 
\end{cases}
\]

We see easily that the function \(g\) fulfills the conditions \(i < j \rightarrow g(i) < g(j)\) and \(m_i \sqsubseteq m'_{f(i)}\). This completes our proof of 1 \(\Rightarrow\) 2.

### B.2 Proof of theorem 2:

Because of theorem 1 and the definition of \(\sqsubseteq\) it is sufficient to show that \(\sqsubseteq_2\) is a quasi-order on \(M_N\).

Let \((m_1, \ldots, m_k) \in M_N\). The function \(\text{Id} : \{1, \ldots, k\} \rightarrow \{1, \ldots, k\}\) with \(\text{Id}(i) = i\) fulfills all required conditions of the definition of \(\sqsubseteq_2\). Therefore we have \((m_1, \ldots, m_k) \sqsubseteq_2 (m_1, \ldots, m_k)\) and \(\sqsubseteq_2\) is reflexive.

Let \((m_1, \ldots, m_k), (m'_1, \ldots, m'_i), (m''_1, \ldots, m''_h) \in M_N\) with \((m_1, \ldots, m_k) \sqsubseteq_2 (m'_1, \ldots, m'_i)\) and \((m'_1, \ldots, m'_i) \sqsubseteq_2 (m''_1, \ldots, m''_h)\). From the definition of \(\sqsubseteq_2\) follows the existence of functions \(f : \{1, \ldots, k\} \rightarrow \{1, \ldots, l\}\) and \(g : \{1, \ldots, l\} \rightarrow \{1, \ldots, h\}\) which fulfills the conditions of these definition.

But then clearly the function \(g \circ f : \{1, \ldots, k\} \rightarrow \{1, \ldots, h\}\) also fulfills these conditions. Therefore we have \((m_1, \ldots, m_k) \sqsubseteq_2 (m''_1, \ldots, m''_h)\) and \(\sqsubseteq_2\) is also transitive.

That means \(\sqsubseteq_2\) (and also \(\sqsubseteq\)) is reflexive and transitive and therefore a quasi-order on \(M_N\). This completes the proof.
# C Tables of problems

## C.1 Experiment 1\(^{13}\)

<table>
<thead>
<tr>
<th>Number</th>
<th>Type</th>
<th>Position</th>
<th>Solution</th>
<th>Reference/motive</th>
</tr>
</thead>
</table>
| 1      | \(\{D,G,P,F\}\) | White: Ka7 Qh3 Re5 Ktd6  
Black: Kh8 Qg8 Rg8 Bf7 Ph7 | 1. Rg5 Qf6  
2. Qc3 Qc3  
3. Ktf7 mate | |
| 2      | \(\{G,P,F\}\) | White: Kh2 Bf3 Kth5 Pg3,g7  
Black: Kh7 Qe6 Ph3 | 1. g8Q+ Kg8  
2. Bd5 Qd5  
3. Kt8+ | |
| 3      | \(\{D,G,P\}\) | White: Kg1 Qe2 Re1 Bg6,h2 Pf2  
Black: Kf5 Qb7 Rg8 Be7,h3 Pg7,f8 | 1. Qe7+ Qe7  
2. Bd6 Qd6  
3. Re8 mate | |
| 4      | \(\{D,P,F\}\) | White: Kg1 Qc2 Rd2 Bb1 Ktf8 Pb2,c6,g2  
Black: Kd8 Qg7 Rd6 Be4 Ktd3 Pb7,e7 | 1. cb: Bb7  
2. Qd3+: Rd3  
3. Kte5+ | |
| 5      | \(\{D,G,F\}\) | White: Ka2 Qf4 Be3 Pb2,b3,h3,c7  
Black: Ka5 Qe7 Ktb6 Pa6,b5,c5,b4,h4 | 1. Qb4+: cb:  
2. Bb6+: Kb6  
3. c8Kt+ | |
| 6      | \(\{G,P\}\) | White: Kf1 Qa6 Re1 Kth3 Pg2,f2,d4  
Black: Ke8 Qd6 Rh8 Kt6e Pe6,f7,g7 | 1. d5 Qd5  
2. Qa6+ arbitrary  
3. Qe6+/-Qd5/Qh8: | |
| 7      | \(\{D,F\}\) | White: Kd6 Ktf5 Pe7  
Black: Kf7 Kg4 Ph7 | 1. Kth6+: Kth6:  
2. Ke2 arbitrary  
3. e8Q | |
| 8      | \(\{G,F\}\) | White: Ke6 Ba6 Kte6 Pe4  
Black: Ke8 Pe7,h2 | 1. Be2 h1Q  
2. Bh5+: Qh5:  
3. Kt7g7+ | Maiselis & Judowitsch (1966) |
| 9      | \(\{D,P\}\) | White: Kh2 Bb6 Pf3,g2  
Black: Kt4 Re2 Ph7,h5,g5 | 1. Bc7 Rg2:+  
2. Kg2: arbitrary  
3. Bd8/f2 mate | |
| 10     | \(\{P,F\}\) | White: Kf3 Re6 Kte5 Pg6  
Black: Kg8 Rd4 Be7 Pf4 | 1. Rc8+ Kg7  
2. Rc7 Kf8  
| 11     | \(\{D,G\}\) | White: Kh2 Qd1 Re2 Pd7,f2,h4  
Black: Kg8 Qb5 Rd8 Pa4,g7,h7 | 1. Re8+ Re8:  
2. Qd5+: Qd5:  
3. deQ mate | |
| 12     | \(\{F\}\) | White: Kf2 Kte8,f7 Pd3  
Black: Kf4 Re6 Pd4 | 1. Ktc7 Rg6/c6  
2. Ktd5+ arbitrary  
| 13     | \(\{P\}\) | White: Kf1 Qh6 Re1 Bf6  
Black: Kd8 Qa4 Rb7 Be7 P7 | 1. Qh8+ Qe8  
2. Rd1+ Rd7  
3. Be7+ pin | |
| 14     | \(\{G\}\) | White: Kb5 Qd7 Pa7  
Black: Ka8 Bb4 Pa2,c2 | 1. Kb6 Ba5+/-c5+  
2. Ka6/c6 arbitrary  
3. Qb7/c5 mate guidance | Speckmann (1958) |
| 15     | \(\{D\}\) | White: Kd8 Bb7 Pc7  
Black: Kd6 Be6 Pf6,a6 | 1. Be8 Bd5  
2. Bf5: Bb7  
3. Be4 deflection | |

\(^{13}\)Problems without a reference were constructed by Bernd Hierholz (Chess-Club Ladenburg, FRG) in 1988.
### C.2 Experiment II

<table>
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<tr>
<th>Number</th>
<th>Type</th>
<th>Position</th>
<th>solution</th>
<th>alternative</th>
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</thead>
</table>
| 1      | (S)  | White: Ka1 Qf5  
        |        | Black: Kh8 Pg6 | 1. Q arbitrary  
        |        | except of Qg6;Qh5 | |
| 2      | (G,S)| White: Kh1 Bf2  
        |        | Black: Kc7 Qe5 | 1. Bg3 Qg3:  
        |        | stale-mate | |
| 3      | (E,G,S)| White: Kh1 Bf2 Ktc3  
        |        | Black: Kc7 Qf4 Pe6 | 1. Ktd5+ ed5:  
        |        | 2. Bg3 Qg3:  
        |        | stale-mate | |
| 4      | (E,E,G,S)| White: Ka1 Bc2 Ktf3,d7  
        |        | Black: Kf7 Qc4 Pf6,d6 | 1. Kte5+ de5/fe5:  
        |        | 2. Kte5+ fe5/fe5:  
        |        | 3. Bh3 Qb3:  
        |        | stale-mate | |
| 5      | (G,S)| White: Ka3 Pb2  
        |        | Black: Kc6 Rh2 Pa4,b5,c4 | 1. b4+ ab3ep/ch3ep  
        |        | stale-mate | |
| 6      | (G,C,S)| White: Kh3 Bg4 Pg2  
        |        | Black: Kg6 Rc2 Ph4,g5,f4 | 1. Bf6+ Kf5:  
        |        | 2. g4+ hg3ep/fg3ep  
        |        | stale-mate | |
| 7      | (T,S)| White: Kh5 Pf7  
        |        | Black: Kh7 | 1. f8R  
        |        | win | |
| 8      | (E,S)| White: Kh1 Ktd2  
        |        | Black: Kd6 Qg3 Pd5 | 1. Kte4+ de4:  
        |        | stale-mate | |
| 9      | (G,E,S)| White: Ka1 Bc2 Kte2  
        |        | Black: Ke6 Qc4 Pe5 | 1. Bb3 Qb3:  
        |        | 2. Ktd4+ ed4:  
        |        | stale-mate | |
| 10     | (F) | White: Kb2 Kt3  
        |        | Black: Kf7 Qc6 | 1. Kte5+ draw | |
| 11     | (G,F)| White: Kh1 Be3 Ktc3  
        |        | Black: Kc7 Qe5 | 1. Bf4 Qf4:  
        |        | 2. Ktd5+ draw | |
| 12     | (G,F,F)| White: Kg1 Bc6 Kte4,g4  
        |        | Black: Kg8 Qe6 Pg7 | 1. Bd5 Qd5:  
        |        | 2. Ktf6+ gf6:  
        |        | 3. Ktf6+ draw | |
| 13     | (G,G,F,F)| White: Kb1 Bg3 Kh4,e7 Pf7,e2  
        |        | Black: Kh8 Qf6 Bg7 Ph7 | 1. f8Q(R)+ Bf8:  
        |        | 2. Be5 Qe5:  
        |        | 3. Ktg6+ hg6:  
        |        | 4. Ktg6+ draw | |
| 14     | (G,G,F)| White: Kb1 Bg3 Kth4 Pf7,e2  
        |        | Black: Kh8 Qf6 Bg7 | 1. f8Q(R)+ Bf8:  
        |        | 2. Be5 Qe5:  
        |        | 3. Ktg6+ draw | |
| 15     | (F,F)| White: Kb2 Kte3,c3  
        |        | Black: Kc7 Qf4 Pe6 | 1. Ktd5+ ed5:  
        |        | 2. Ktd5+ draw | |
| 16     | (T,F)| White: Kc1 Pf7,b2  
        |        | Black: Kh7 Qd7 | 1. f8Kt+ win | |
| 17     | (T,F,F)| White: Kb1 Kte4 Pf7  
        |        | Black: Kh7 Qd7 Bg7 | 1. f8Kt+ Bf8:  
        |        | 2. Ktf6+ draw | |
C.3  "Dummy Problems" of Experiment II

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| D1     | White: Kg3 Rh1  
         | Black: Kb3 Qb7 | 1. Rb1: arbitrary  
         |           | White wins      |
| D2     | White: Kc6 Ra1  
         | Black: Kh8    | 1. Rh1: mate      |
| D3     | White: Ka1 Rg8 Pa7  
         | Black: Kh4    | 1. a8Q arbitrary  
         |           | 2. Qh1 mate      |
## D  Experiment I: solution times

The last row ($\bar{\Sigma}$) shows the average solution times for each problem. The $\Sigma$-column contains the total solution times and the $\bar{\Sigma}$-column the average solution times for each subject.

| Subject | DGPF | GPF | DGP | DPF | DGF | GP | DP | GP | DP | PF | DG | F | P | G | D | $\Sigma$ | $\bar{\Sigma}$ |
|---------|------|-----|-----|-----|-----|----|----|----|----|----|----|---|---|---|---|---|-------|--------|
| 1       | 180  | 105 | 180 | 885 | 120 | 420| 39 | 210| 75 | 345| 60 | 165| 180| 120| 285| 3360  | 224     |
| 2       | 195  | 105 | 45  | 465 | 90  | 255| 60 | 75 | 120| 165| 120| 195| 75 | 240| 105| 2310  | 154     |
| 3       | 285  | 60  | 90  | 660 | 75  | 255| 165| 90 | 75 | 225| 270| 930| 135| 240| 465 | 4020  | 268     |
| 4       | 360  | 75  | 120 | 420 | 75  | 705| 75 | 105| 75 | 285| 75 | 165| 75 | 105| 120| 2835  | 189     |
| 5       | 420  | 30  | 30  | 780 | 60  | 60 | 30 | 30 | 75 | 330| 45 | 210| 90 | 180| 30 | 2400  | 160     |
| 6       | 825  | 135 | 60  | 1396| 60  | 210| 165| 240| 180| 930| 165| 375| 60 | 360| 375 | 5535  | 369     |
| 7       | 1590 | 120 | 135 | 375 | 150 | 660| 135| 225| 195| 420| 135| 240| 180| 240| 180 | 4920  | 328     |
| 8       | 300  | 210 | 150 | 600 | 540 | 270| 75 | 210| 240| 240| 120| 450| 330| 315| 660 | 4710  | 314     |
| 9       | 645  | 855 | 720 | 630 | 660 | 225| 420| 720| 480| 600| 600| 615| 930| 765| 480 | 9345  | 623     |
| 10      | 1050 | 480 | 840 | 1260| 420 | 600| 330| 630| 660| 1410| 350| 300| 1290| 660| 660 | 10890 | 726     |
| 11      | 1125 | 120 | 255 | 420 | 135 | 180| 390| 150| 225| 1860| 420| 420| 180| 210| 90  | 6180  | 412     |
| 12      | 1650 | 465 | 135 | 600 | 180 | 1680| 300| 600| 420| 350| 300| 780| 180| 300| 225 | 8166  | 544     |
| 13      | 1560 | 300 | 300 | 1920| 360 | 855| 120| 300| 180| 900| 165| 600| 525| 360| 645 | 9060  | 606     |

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Experiment II: instructions for the subjects


Zunächst wird Ihnen die Stellung 90 Sekunden lang gezeigt, ohne daß Sie die Möglichkeit haben zu ziehen. Bitte überlegen Sie sich während dieser Zeit, welcher Zug oder welche Zugkombination mit so wenig Zügen wie möglich für Weiß zum besten Ergebnis führt!


Stellen Sie bitte während des Versuchs keine Fragen an den Versuchsleiter. Falls Sie noch Fragen haben sollten, fragen Sie bitte jetzt. Um den
Versuchsdurchgang zu starten, bewegen Sie bitte den Pfeil mit der Maus auf "Start" und drücken die linke Taste. Wir wünschen Ihnen viel Erfolg.

**Text presented during “thinking time”** Bitte nutzen Sie nun die Bedenkzeit von 90 Sekunden, um zu überlegen, mit welchem Zug oder welcher Zugkombination Sie — mit so wenig Zügen wie möglich — das beste Ergebnis für Weiß erzielen können.


F Experiment II: Overview of individual results

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DISKUSIONS-PAPIER Nr. 17: Gundlach, H.: Inventarium der älteren Experimentalapparate im Psychologischen Institut Heidelberg sowie einige historische Bemerkungen. 1978


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