Optimization and the Psychology of Human Decision Making

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Outline

Introduction

Mathematical formulation

Reformulations

Using Optimization as an Analysis Tool

Algorithm

Conclusions and Outlook
Goals of psychologists

- Research complex problem solving of human beings
  - Want to understand how external factors influence thinking
    - Example: positive or negative feedback
    - Example: stress
    - Example: learning effects
  - Approach: use computer-based test scenarios
    - Evaluate performance and correlate it to attributes
      - Example: proband’s capacity of emotion regulation
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- High-order cognitive process
- Complexity stems from:
  - coupling, nonlinearities, dynamics, intransparency, ... 
- Psychologists work since $\approx 100$ years on understanding
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- Psychologists work since \( \approx 100 \) years on understanding
  - [Ewert & Lambert, 1932]: disk problem

- Since 70s/80s: also use computer simulations
Measure capacity to solve complex problems

- Measure proband’s performance
  - Performance in a round based test scenario
  - Tailorshop developed in the 80s by Dörner
  - Referenced in many studies and books by now
  - Collect data from probands:
    - Quantified emotions: own statements
    - Standardized tests to classify probands according to groups, e.g., good or poor emotional regulation
    - Quantified emotions: observation of study leader
    - Quantified emotions: video analysis
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The tailorshop

- Round based decision making
- How to produce, distribute, and sell shirts

Manufacturing → Logistics → Sales

- Goal: maximize profit after 12 months
## Hier der Zustand Ihres Ladens am Ende von Monat 0

<table>
<thead>
<tr>
<th>Flüssigkapital</th>
<th>165775</th>
<th>Gesamtkapital (Bilanz)</th>
<th>250691</th>
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<tbody>
<tr>
<td>verkaufte Hemden</td>
<td>407</td>
<td>Nachfrage (aktuell)</td>
<td>767</td>
</tr>
<tr>
<td>Rohmaterial: Preis</td>
<td>4</td>
<td>Rohmaterial: im Lager</td>
<td>16</td>
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<tr>
<td>fertige Hemden im Lager</td>
<td>81</td>
<td>50-Hemden-Maschinen</td>
<td>10</td>
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<tr>
<td>Arbeiter für 50er</td>
<td>8</td>
<td>100-Hemden-Maschinen</td>
<td>0</td>
</tr>
<tr>
<td>Arbeiter für 100er</td>
<td>0</td>
<td>Reparatur &amp; Service</td>
<td>1200</td>
</tr>
<tr>
<td>Lohn pro Arbeiter</td>
<td>1080</td>
<td>Sozialkosten pro Arbeiter</td>
<td>50</td>
</tr>
<tr>
<td>Preis pro Hemd</td>
<td>52</td>
<td>Ausgaben für Werbung</td>
<td>2800</td>
</tr>
<tr>
<td>Anzahl der Lieferwagen</td>
<td>1</td>
<td>Geschäftslage</td>
<td>Cityrand</td>
</tr>
<tr>
<td>Arbeitszufriedenheit in %</td>
<td>57.7</td>
<td>Maschinen-Schäden in %</td>
<td>5.9</td>
</tr>
<tr>
<td>Produktionsausfall in %</td>
<td>0.0</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### Maßnahmen für Monat 1

- R = Rohmaterial einkaufen
- W = Kosten für Werbung ändern
- M = Maschinen (ver)kaufen, tauschen
- L = Lohn pro Arbeiter ändern
- G = Geschäftslage wechseln
- H = Hemdenpreis ändern
- A = Arbeiter einstellen oder entlassen
- I = Instandhaltung, Reparatur/Service
- S = Sozialkosten pro Arbeiter ändern
- T = Lieferwagen kaufen oder verkaufen
- D = Informationen aus der Datenbank
- E = Ende der Eingriffe für diesen Monat
So what is missing?

► Main motivation for simple test scenarios
  ► Optimal solution is known
  ► Proband’s performance is easy to analyze

► More complex scenarios
  ► Optimal solution is NOT known
  ► Performance only comparable among probands
    or isolated indices, e.g., advance in overall capital
  ► Hard to say when and what the wrong decisions were

► Is it possible to have a detailed (and correct) analysis?
  ► Yes. Need to formulate optimization problem!
So what is missing?

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- Is it possible to have a detailed (and correct) analysis?
  - Yes. Need to formulate optimization problem!
Modeling - what was available?

▶ Heuristic descriptions
▶ GWBasic source code
GUI used for tests
GUI used for tests
GUI used for tests
Available GW Basic source code – extract

2650  ZA=.5+((LO-850)/550)+SM/800:IF ZA>ZM THEN:ZA=ZM
2660  SK=SM*(N1+N2):KA=KA-SK
2670  X=A1:IF N1<X THEN:X=N1
2680  Y=A2:IF N2<Y THEN:Y=N2
2690  PM=X*(MA+RND*4-2)+Y*(MA*2+RND*6-3):PM=PM*ABS(ZA)^.5
2700  X=PM:IF RL<X THEN:X=RL
2710  PA=X:HL=HL+PA:RL=RL-PA:KA=KA-(PA*1)-(RL*.5)
2720  NA=(NA/2+280)*1.25*2.7181^(-(PH^2)/4250):KA=KA-HL
2730  X=NA:IF HL<X THEN:X=HL
2740  VH=X:HL=HL-VH:KA=KA+VH*PH
2750  KA=KA-WE
2760  X1=WE/5:IF X1>NM THEN:X1=NM
2770  KA=KA-LW*500:X1=X1+LW*100
2780  KA=KA-GL*2000
2790  X=0:IF GL=.5 THEN:X=.1:ELSE IF GL=1 THEN:X=.2
2800  X1=X1+X1*X
2810  NA=X1+(RND*100-50)
2820  RP=2+(RND*6.5)
2830  MA=MA-.1*MA+(RS/(A1+A2*1E-08))*0.017
2840  IF MA>MM THEN:MA=MM
2850  KA=KA-RS
Observations

- **Nonlinear**

  2720 \( NA = \frac{NA}{2} + 280 \times 1.25 \times 2.7181^{-\left(\frac{PH^2}{4250}\right)} \)
Observations

- **Nonlinear**
  
  \[ NA = \left(\frac{NA}{2} + 280\right) \times 1.25 \times 2.7181^{\left(\frac{-PH^2}{4250}\right)} \]

- **Integer variables**
  
  \[ X = 0: \text{IF} \ GL = .5 \ \text{THEN}: X = .1: \text{ELSE} \text{IF} \ GL = 1 \ \text{THEN}: X = .2 \]
Observations

▶ Nonlinear

2720 \[ NA = \frac{(NA/2 + 280) \times 1.25 \times 2.7181^{-(PH^2)/4250}}{2} \]

▶ Integer variables

2790 \[ X = 0: \text{IF } GL = 0.5 \text{ THEN: } X = 0.1: \text{ELSE IF } GL = 1 \text{ THEN: } X = 0.2 \]

▶ Random values \( \xi \)

2810 \[ NA = X + (RND \times 100 - 50) \]
Observations

- **Nonlinear**
  
  \[ NA = \frac{NA}{2} + 280 \times 1.25 \times 2.7181^{\left(\frac{-(PH^2)}{4250}\right)} \]

- **Integer variables**
  
  \[ X = 0: \text{IF } GL = 0.5 \text{ THEN: } X = 0.1: \text{ELSE IF } GL = 1 \text{ THEN: } X = 0.2 \]

- **Random values** \(\xi\)
  
  \[ NA = X_1 + (RND \times 100 - 50) \]

- **Nondifferentiable**
  
  \[ ZA = \frac{.5 + ((LO - 850)/550) + SM/800}{1}: \text{IF ZA} > ZM \text{ THEN: } ZA = ZM \]
Observations

- **Nonlinear**
  
  \[ NA = (NA/2+280) \times 1.25 \times 2.7181^{\left(-\left(\text{PH}^2\right)/4250\right)} \]

- **Integer variables**
  
  \[ X = 0 \quad \text{IF} \quad GL = .5 \quad \text{THEN} \quad X = .1 \quad \text{ELSE} \quad \text{IF} \quad GL = 1 \quad \text{THEN} \quad X = .2 \]

- **Random values \( \xi \)**
  
  \[ NA = X1 + (RND \times 100 - 50) \]

- **Nondifferentiable**
  
  \[ ZA = .5 + \left(\frac{LO-850}{550}\right) + SM/800 \quad \text{IF} \quad ZA > ZM \quad \text{THEN} \quad ZA = ZM \]

- **Sometimes variable time \( k \), sometimes already updated**
  
  \[ PM = X \times (MA + RND \times 4 - 2) + Y \times (MA \times 2 + RND \times 6 - 3) \quad \text{PM} = PM \times (ABS(ZA)^{.5}) \]
  
  \[ X = PM \quad \text{IF} \quad RL < X \quad \text{THEN} \quad X = RL \]
  
  \[ PA = X \quad \text{HL} = HL + PA \quad \text{RL} = RL - PA \quad \text{KA} = KA - (PA \times 1) - (RL \times .5) \]
Abstract optimization model

- Dynamic model with discrete time $k = 0 \ldots N$
Abstract optimization model

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- Decisions $u_k = u(k)$ and states $x_k = x(k)$
Abstract optimization model

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- Goal: find decisions $u_k$
  to maximize objective function of $x_N$
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  to maximize objective function of $x_N$

$$\max_{x,u} \quad F(x_N)$$

$$\begin{align*}
\text{s.t.} \quad & x_{k+1} = G(x_k, x_{k+1}, u_k, p, \xi), \quad k = 0 \ldots N - 1, \\
& 0 \leq H(x_k, x_{k+1}, u_k, p), \quad k = 0 \ldots N - 1, \\
& u_k \in \Omega, \quad k = 0 \ldots N - 1.
\end{align*}$$
### Control functions $u_k$

<table>
<thead>
<tr>
<th>Decision</th>
<th>$low \leq u_k \leq up$</th>
</tr>
</thead>
<tbody>
<tr>
<td>advertisement</td>
<td>$0 \leq WE \leq \infty$</td>
</tr>
<tr>
<td>shirt price</td>
<td>$10 \leq PH \leq 100$</td>
</tr>
<tr>
<td>buy raw material</td>
<td>$0 \leq \Delta RL \leq \infty$</td>
</tr>
<tr>
<td>workers 50</td>
<td>$-A_1 \leq \Delta A_1 \leq \infty$</td>
</tr>
<tr>
<td>workers 100</td>
<td>$-A_2 \leq \Delta A_2 \leq \infty$</td>
</tr>
<tr>
<td>buy machines 50</td>
<td>$0 \leq \Delta M_1 \leq \infty$</td>
</tr>
<tr>
<td>buy machines 100</td>
<td>$0 \leq \Delta M_2 \leq \max(0, MA - 35) \cdot \infty$</td>
</tr>
<tr>
<td>sell machines 50</td>
<td>$0 \leq \delta M_1 \leq M_1$</td>
</tr>
<tr>
<td>sell machines 100</td>
<td>$0 \leq \delta M_2 \leq M_2$</td>
</tr>
<tr>
<td>maintenance</td>
<td>$0 \leq RS \leq \infty$</td>
</tr>
<tr>
<td>wages</td>
<td>$850 \leq LO \leq \infty$</td>
</tr>
<tr>
<td>social spenses</td>
<td>$0 \leq SM \leq \infty$</td>
</tr>
<tr>
<td>buy vans</td>
<td>$0 \leq \Delta LW \leq \infty$</td>
</tr>
<tr>
<td>sell vans</td>
<td>$0 \leq \delta LW \leq LW$</td>
</tr>
<tr>
<td>Choose site</td>
<td>$GL \in {c, r, v}$</td>
</tr>
</tbody>
</table>
### State variables $x_{k+1}$ and $x_k$

<table>
<thead>
<tr>
<th>State</th>
<th>$x_{k+1}$</th>
<th>$G(x_k, x_{k+1}, u_k, p, \xi)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>machines 50</td>
<td>$M_1$</td>
<td>$M_1 + \Delta M_1 - \delta M_1$</td>
</tr>
<tr>
<td>machines 100</td>
<td>$M_2$</td>
<td>$M_2 + \Delta M_2 - \delta M_2$</td>
</tr>
<tr>
<td>workers 50</td>
<td>$A_1$</td>
<td>$A_1 + \Delta A_1$</td>
</tr>
<tr>
<td>workers 100</td>
<td>$A_2$</td>
<td>$A_2 + \Delta A_2$</td>
</tr>
<tr>
<td>demand</td>
<td>$NA$</td>
<td>$100\xi - 50$</td>
</tr>
<tr>
<td>vans</td>
<td>$LW$</td>
<td>$LW + \Delta LW - \delta LW$</td>
</tr>
<tr>
<td>shirts sales</td>
<td>$VH$</td>
<td>$\min(HL, \frac{5}{4} \left(\frac{NA}{2} + 280\right) \cdot 2.7181 - \frac{PH^2}{4250})$</td>
</tr>
<tr>
<td>shirts stock</td>
<td>$HL$</td>
<td>$HL + PA - VH$</td>
</tr>
<tr>
<td>possible production</td>
<td>$PM$</td>
<td>$(\min(A_1, M_1)(MA + 4\xi - 2) + \min(A_2, M_2)(2MA + 6\xi - 3)) \cdot</td>
</tr>
<tr>
<td>actual production</td>
<td>$PA$</td>
<td>$\min(PM, RL + \Delta RL)$</td>
</tr>
<tr>
<td>material price</td>
<td>$RP$</td>
<td>$2 + 6.5\xi$</td>
</tr>
<tr>
<td>material stock</td>
<td>$RL$</td>
<td>$RL + \Delta RL - PA$</td>
</tr>
<tr>
<td>satisfaction</td>
<td>$ZA$</td>
<td>$\min\left(ZM, \frac{1}{2} + \frac{LO-850}{550} + \frac{SM}{800}\right)$</td>
</tr>
<tr>
<td>machine capacity</td>
<td>$MA$</td>
<td>$\min\left(MM, 0.9MA + 0.017 \frac{RS}{M_1 + 10^{-8}M_2}\right)$</td>
</tr>
</tbody>
</table>
State variables: money

\[
UK = KA + VH \cdot PH - RP \cdot \Delta RL \\
-10000\Delta M_1 + 8000 \frac{MA}{MM} \delta M_1 - 20000\Delta M_2 + 16000 \frac{MA}{MM} \delta M_2 \\
-SK - WE - RS - (A_1 + A_2) \cdot LO \\
-PA - \frac{1}{2} RL - (HL + PA) \\
-10000 \cdot \Delta LW + (8000 - 100k) \cdot \delta LW - 500 LW \\
\begin{cases}
2000 & \text{if } GL = c \\
1000 & \text{if } GL = r \\
500 & \text{if } GL = v
\end{cases}
\]

\[
KA = UK \left(1 + \begin{cases}
GZ & \text{if } UK \geq 0 \\
SZ & \text{if } UK < 0
\end{cases}\right)
\]

Goal: maximize \( L_N \):

\[
L = KA + \frac{MA}{MM} (8000M_1 + 16000M_2) \\
+(8000 - 100k) \cdot LW + 2RL + 20HL
\]
Fixed initial values $x_0$ and parameters $p$ 

<table>
<thead>
<tr>
<th>State</th>
<th>$x_k$</th>
<th>$x_0 =$</th>
</tr>
</thead>
<tbody>
<tr>
<td>machines 50</td>
<td>$M_1$</td>
<td>10</td>
</tr>
<tr>
<td>machines 100</td>
<td>$M_2$</td>
<td>0</td>
</tr>
<tr>
<td>workers 50</td>
<td>$A_1$</td>
<td>8</td>
</tr>
<tr>
<td>workers 100</td>
<td>$A_2$</td>
<td>0</td>
</tr>
<tr>
<td>demand</td>
<td>$NA$</td>
<td>766.636</td>
</tr>
<tr>
<td>material price</td>
<td>$RP$</td>
<td>3.9936</td>
</tr>
<tr>
<td>material stock</td>
<td>$RL$</td>
<td>16.06787</td>
</tr>
<tr>
<td>shirts stock</td>
<td>$HL$</td>
<td>80.7164</td>
</tr>
<tr>
<td>machine capacity</td>
<td>$MA$</td>
<td>47.04</td>
</tr>
<tr>
<td>cash</td>
<td>$KA$</td>
<td>165774.66</td>
</tr>
<tr>
<td>vans</td>
<td>$LW$</td>
<td>1</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Parameter</th>
<th>$p$</th>
<th>$p =$</th>
</tr>
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<tbody>
<tr>
<td>maximum demand</td>
<td>$NM$</td>
<td>900</td>
</tr>
<tr>
<td>interest rate</td>
<td>$GZ$</td>
<td>0.0025</td>
</tr>
<tr>
<td>debt rate</td>
<td>$SZ$</td>
<td>0.0066</td>
</tr>
<tr>
<td>maximum machine capacity</td>
<td>$MM$</td>
<td>50</td>
</tr>
<tr>
<td>maximum satisfaction</td>
<td>$ZM$</td>
<td>1.7</td>
</tr>
</tbody>
</table>
Modeling issues

\[
\max_{x,u} \ F(x_N)
\]

s.t. \[x_{k+1} = G(x_k, x_{k+1}, u_k, p, \xi), \quad k = 0 \ldots N - 1,\]
\[0 \leq H(x_k, x_{k+1}, u_k, p), \quad k = 0 \ldots N - 1,\]
\[u_k \in \Omega, \quad k = 0 \ldots N - 1.\]

▶ More realistic modeling (delays, memory effects, ...)
Modeling issues

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\begin{align*}
\max_{x,u} & \quad F(x_N) \\
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Modeling issues

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\end{align*}
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- Modeling errors
- Random values $\xi$
Modeling issues

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- More realistic modeling (delays, memory effects, \ldots)
- Modeling errors
- Random values $\xi$
- Bounds on variables
Modeling issues

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- More realistic modeling (delays, memory effects, \ldots)
- Modeling errors
- Random values $\xi$
- Bounds on variables
- Integer decisions
- $F(\cdot)$, $G(\cdot)$ and $H(\cdot)$ continuously differentiable? Expressions including $\text{if}$, $\text{min}$, or $\text{max}$ are not!
Consistency

- More realistic model only with new study
Consistency

- More realistic model only with new study
- Modelling errors: have to accept and include them

\[ MA = \min \left( MM, 0.9MA + 0.017 \frac{RS}{M_1 + 10^{-8} M_2} \right) \]

\[ \rightarrow RS = \epsilon \] optimal
Consistency

- More realistic model only with new study
- Modelling errors: have to accept and include them

\[ MA = \min \left( MM, 0.9 MA + 0.017 \frac{RS}{M_1 + 10^{-8} M_2} \right) \]

\[ \rightarrow RS = \epsilon \text{ optimal} \]

- Random values \( \xi \)

140 X=RND (-1)

\[ \ldots \]

2810 NA=X1+(RND*100-50)

Random values \( \xi \) can be treated as parameters \( p \)!
## Integer decisions

<table>
<thead>
<tr>
<th>Decision</th>
<th>( \text{low} \leq u_k \leq \text{up} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>advertisement</td>
<td>( 0 \leq WE \leq \infty )</td>
</tr>
<tr>
<td>shirt price</td>
<td>( 10 \leq PH \leq 100 )</td>
</tr>
<tr>
<td>buy raw material</td>
<td>( 0 \leq \Delta RL \leq \infty )</td>
</tr>
<tr>
<td>workers 50</td>
<td>(-A_1 \leq \Delta A_1 \leq \infty )</td>
</tr>
<tr>
<td>workers 100</td>
<td>(-A_2 \leq \Delta A_2 \leq \infty )</td>
</tr>
<tr>
<td>buy machines 50</td>
<td>( 0 \leq \Delta M_1 \leq \infty )</td>
</tr>
<tr>
<td>buy machines 100</td>
<td>( 0 \leq \Delta M_2 \leq \max(0, MA - 35) \cdot \infty )</td>
</tr>
<tr>
<td>sell machines 50</td>
<td>( 0 \leq \delta M_1 \leq M_1 )</td>
</tr>
<tr>
<td>sell machines 100</td>
<td>( 0 \leq \delta M_2 \leq M_2 )</td>
</tr>
<tr>
<td>maintenance</td>
<td>( 0 \leq RS \leq \infty )</td>
</tr>
<tr>
<td>wages</td>
<td>( 850 \leq LO \leq \infty )</td>
</tr>
<tr>
<td>social spenses</td>
<td>( 0 \leq SM \leq \infty )</td>
</tr>
<tr>
<td>buy vans</td>
<td>( 0 \leq \Delta LW \leq \infty )</td>
</tr>
<tr>
<td>sell vans</td>
<td>( 0 \leq \delta LW \leq LW )</td>
</tr>
<tr>
<td>Choose site</td>
<td>( GL \in {c, r, v} )</td>
</tr>
</tbody>
</table>
Bounds

- Optimizer’s intuition: no bounds on variables
  → unbounded solution
Bounds

- Optimizer’s intuition: no bounds on variables → unbounded solution
- Combination of model error and no bound. Demand

\[
NA = a + \left( \min\left( \frac{WE}{5}, NM \right) + 100LW \right) \cdot b
\]

enters into number of shirts sold

\[
VH = \min(HL, \frac{5}{4} \left( \frac{NA}{2} + 280 \right) \cdot 2.7181 - \frac{PH^2}{4250})
\]
Bounds

- Optimizer’s intuition: no bounds on variables
  $\implies$ unbounded solution
- Combination of model error and no bound. Demand
  $$NA = a + \left( \min\left( \frac{WE}{5}, NM \right) + 100LW \right) \cdot b$$

  enters into number of shirts sold

  $$VH = \min(HL, \frac{5}{4} \left( \frac{NA}{2} + 280 \right) \cdot 2.7181^{-\frac{PH^2}{4250}})$$

- Need to include bounds – consistency!
Nondifferentiabilities

\[ \min (ZM, \frac{1}{2} + \frac{LO-850}{550} + \frac{SM}{800}) \]
Nondifferentiabilities

\[ \min (ZM, \frac{1}{2} + \frac{LO - 850}{550} + \frac{SM}{800}) \rightarrow \frac{1}{2} + \frac{LO - 850}{550} + \frac{SM}{800} \leq ZM \]
Nondifferentiabilities

- \( \min \left( ZM, \frac{1}{2} + \frac{LO-850}{550} + \frac{SM}{800} \right) \rightarrow \frac{1}{2} + \frac{LO-850}{550} + \frac{SM}{800} \leq ZM \)
- \( \min(PM, RL + \Delta RL) \rightarrow RL + \Delta RL \leq PM \)
- \( \min(HL, \frac{5}{4} \left( \frac{NA}{2} + 280 \right) \cdot 2.7181^{\frac{PH^2}{4250}}) \)
  \( \rightarrow \frac{5}{4} \left( \frac{NA}{2} + 280 \right) \cdot 2.7181^{\frac{PH^2}{4250}} \leq HL \)
- \( \min\left( \frac{WE}{5}, NM \right) \rightarrow \frac{WE}{5} \leq NM \)
- \( \min\left( MM, 0.9MA + 0.017 \frac{RS}{M_1+10^{-8}M_2} \right) \rightarrow \\
  0.9MA + 0.017 \frac{RS}{M_1+10^{-8}M_2} \leq MM \)
- \( \min(A_1, M_1), \min(A_2, M_2) \rightarrow A_1 \leq M_1, A_2 \leq M_2 \)
- Buy machines (100) only if \( MA > 35 \):
  \( \rightarrow 0 \leq \Delta M_2 \leq \max(0, MA - 35) \cdot \infty \)
  \( \rightarrow MA \geq 36 \)
- \( KA = UK \left( 1 + \begin{cases} 
  GZ & \text{if } UK \geq 0 \\
  SZ & \text{if } UK < 0 
 \end{cases} \right) \)?
Optimization problem

\[
\begin{align*}
\max_{x,u} & \quad F(x_N) \\
\text{s.t.} & \quad x_{k+1} = G(x_k, x_{k+1}, u_k, p), \quad k = 0 \ldots N - 1, \\
& \quad 0 \leq H(x_k, x_{k+1}, u_k, p), \quad k = 0 \ldots N - 1, \\
& \quad u_k \in \Omega, \quad k = 0 \ldots N - 1.
\end{align*}
\]

- 5 continuous control functions
Optimization problem

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- No uncertainty
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\]

- 5 continuous control functions
- 10 integer control functions
- 17 state functions
- No uncertainty
- Differentiable
- Mixed-integer Nonlinear Program (MINLP)
Intermediate summary

- Go from simple test scenarios to complex scenarios
- Determine month(s) $k$ with **bad** decisions
- Do not use progress in objective as currently done!
- Compare optimal solutions at time $k$ and $k + 1$ as measure
- Optimal solutions = solutions of MINLPs
Analysis

- For every data set
  - For every month from 0 to 11
    - Calculate optimal solution for rest of time
    - Store objective value at end time
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![Graph showing the objective function over time](image-url)
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  ▶ For every month from 0 to 11
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Objective of proband vs. potential (in black)
Objective of proband vs. potential (in black)

Analysis function 3

(x\times10^5)

Objective vs. how much is still possible

Month
Objective of proband vs. potential (in black)
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Analysis function 3

Objective vs. how much is still possible

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Further analysis

- Determine WHICH decision was really bad
- Can evaluate the derivative
- No need: already know the optimal solution
  - Look at \((u^*, x^*) - (u^P, x^P)\)
Further analysis

- Determine WHICH decision was really bad
- Can evaluate the derivative
- No need: already know the optimal solution
  - Look at \((u^*, x^*) - (u^p, x^p)\)

Better:
- Solve problem from \(k + 1\) to \(N\) as before
- Add constraints \(u_{k,i} = u_{k,i}^p\), calculate Lagrange multipliers
- Shadow prices: how much does decision \(i\) at time \(k\) cost?
Solver

- Modeling done with AMPL
- Automatization of interfaces
Solver

- Modeling done with AMPL
- Automatization of interfaces
- Structure exploiting interior point method
- IPOPT (Wächter et al.)
- Bonmin (Bonami et al.)

Needed to solve $80 \cdot 12$ optimization problems

- Runtimes each on notebook
  - relaxed: $< 1$ sec.
  - integer: $\approx 3$ min.

Without hotstarts or advanced numerical techniques

No multiple local minima found so far
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Conclusions

- Computer based micro worlds used to understand human complex problem solving
- Modelled one of the most famous ones (tailorshop) as an optimization problem
- By solving series of optimization problems get valuable additional information
- Important: good modelling, exploiting structure
Outlook

▶ Apply new analysis tool to interesting test sets
▶ Apply statistical tools
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- Apply statistical tools
- Improve numerics
  - Warmstarts
  - Initial value embedding
- Will allow for online feedback

From my point of view this is a sensational breakthrough in psychology. This new analysis tool will revolutionize the research field!
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Thank you very much for your attention!

Questions as complex problems for me?
Add constraint: capital ≥ 0
Add constraint: capital $\geq 0$
Add constraint: capital $\geq$ min capital of probands
Add constraint: \( \text{capital} \geq \text{min capital of probands} \)
Add constraint: capital $\geq -10^{10}$
Add constraint: capital \( \geq -10^{10} \)
Fix # of vans to proband’s choice
Fix # of vans to proband’s choice